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MODEL REPRESENTATION OF THE ULTIMATE STRAIN DURING CREEP

M. D. Dacheva, A. M. Lokoshchenko, and S. A. Shesterikov

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Within the framework of the Yu. N. Rabotnov conception of the equation of state [1], we consider a description of strain processes under conditions when cumulative damage ω is governing.

We use the version of the equation of state proposed in [2]

$$d\varepsilon/dt = G'(s)ds/dt + F(s); \tag{1}$$

$$d\omega/dt = g'(s)ds/dt + f(s). \tag{2}$$

Equations (1) and (2) determine the change in the total strain ε and the damage ω in the time t up to rupture $t = t^*$. The prime denotes differentiation with respect to the effective stress s. The functions G'(s), F(s), g'(s), and f(s) in (1) and (2) grow monotonically as s increases.

Let a constant tensile force be applied to a specimen and cause a stress σ_0 and a small strain ε. The effective stress in (1) and (2) is a function of the parameter ω. We consider two versions of this function. In the first, we take the dependence

$$s = \sigma_0 \exp \omega, \tag{3}$$

which, for small strain, agrees with the dependence proposed in [3]. In the second version we use the function

$$s = \sigma_0/(1 - \omega), \tag{4}$$

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introduced in [1].

Let us initially examine the first version of this dependence $s(\omega)$. We find the value of the effective stress s_0 at the loading time ($t = +0$) from (2) and (3):

$$s_0 = \sigma_0 \exp g(s_0). \tag{5}$$

It is shown in [3] that by using the condition $ds/dt \to +\infty$ an equation can be obtained for the limit value of the effective stress $s^* = s(t^*)$.

$$[sg'(s)]|_{s^*} = 1. \tag{6}$$

It follows from (6) that the limit value s^* depends only on the form of the material function $g(s)$ and remains invariant for different values of σ_0. The rupture time $t = t^*$ and its corresponding limit values of the strain $\varepsilon(t^*) = \varepsilon^*$ and damage $\omega(t^*) = \omega^*$ are determined as follows from (1)-(3):

$$t^* = \int_{s_0}^{s^*} K ds, \quad \varepsilon^* = G(s^*) + \int_{s_0}^{s^*} KF ds, \quad \omega^* = g(s^*) + \int_{s_0}^{s^*} Kf ds, \quad K = \frac{1 - sg}{sf}. \tag{7}$$

Let us consider the dependence of the limit values of the strain ε^* and damage ω^* on the stress σ_0. Only the lower limits of the integrals depend on σ_0 in the relation (7), where $s_0(\sigma_0)$ is an increasing function according to (5) and (6). All the remaining quantities in (7) are determined by the material properties and are independent of the stress σ_0. Consequently, an increase in the stress results in a diminution in the limit values t^*, ε^*, and ω^*. Therefore, the model (1) and (2) permits description of test results in which a monotonically decreasing dependence of the limit values of the strain ε^* and the damage ω^* on the nominal value σ_0 is observed by utilizing the relationship between the nominal σ_0 and the effective stress s in the form (3) for any form of the four material functions being used.

Let us turn to the second version of the dependence $s(\omega)$. We consider the system of equations (1), (2), and (4). We show that introduction of the function $s(\omega)$ in the form (4) permits, under certain conditions, obtaining a qualitatively new nonmonotonic nature of the dependence $\varepsilon^*(\sigma_0)$ while conserving the same decreasing dependence $\omega^*(\sigma_0)$. We consider the change in the effective stress s in the time t. At the initial time we obtain the initial value s_0 from (2) and (4):

$$s_0 = \sigma_0/(1 - g(s_0)). \tag{8}$$

Differentiating (4) with respect to time and using (2), we obtain

$$\frac{ds}{dt} = \frac{\sigma_0}{(1-\omega)^2}\frac{d\omega}{dt} = \frac{fs^2}{\sigma_0 - s^2 g'}. \tag{9}$$

The limiting value s^* is determined from the condition $ds/dt \to +\infty$:

$$(s^2 g')|_{s^*} = \sigma_0. \tag{10}$$

It hence follows that the value of s^* increases as σ_0 increases. Eliminating ds/dt from (2) and (9), we obtain

$$\frac{d\omega}{dt} = \frac{f(1-\omega)^2}{(1-\omega)^2 - \sigma_0 g'}, \quad \omega^* = 1 - \sqrt{\sigma_0 g'(s^*)}. \tag{11}$$

There results from (11) that the values of ω^* is always less than 1, where the dependence $\omega^*(\sigma_0)$ is monotonically decreasing in nature. To obtain ε^* we eliminate ds/dt from (1) and (9):

$$\varepsilon^* = G(s^*) + \int_{s_0}^{s^*} \frac{(\sigma_0 - s^2 g')F}{fs^2} ds. \tag{12}$$

In the general case it is impossible to estimate the dependence of ε^* on σ_0. We assume that all the material functions in (1) and (2) are power laws:

$$G' = As^{m_1}, \quad F = Bs^{m_2}, \quad g' = as^{n_1}, \quad f = bs^{n_2}. \tag{13}$$

It is natural to consider that $m_1 > 0$, $n_1 > 0$, $m_2 > 1$, $n_2 > 1$. For convenience, we shall use dimensionless variables. We refer the stresses σ_0 and s and the time t to such quantities that (1), with (13) taken into account, would have coefficients one in both components. Be-

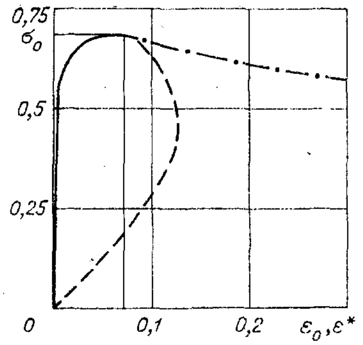

Fig. 1

low we understand by σ_0 and s everywhere, the corresponding dimensionless stresses that equal the true values referred to

$$R = A^{-\frac{1}{m_1+1}}.$$

We also introduce the dimensionless time $\tau = BR^{m_2}t$. In these variables, by taking (13) into account, (1) and (2) take the form

$$\frac{d\varepsilon}{d\tau} = s^{m_1}\frac{ds}{d\tau} + s^{m_2}, \quad \frac{d\omega}{d\tau} = Cs^{n_1}\frac{ds}{dt} + Ds^{n_2}, \tag{14}$$

where $C = aR^{n_1+1}$ and $D = (b/B)R^{n_2-m_2}$. The initial s_0 and limit s^* values are related to σ_0 as follows according to (8) and (10)

$$\sigma_0 = s_0 - \frac{C}{n_1+1}s_0^{n_1+2}, \quad s^* = \left(\frac{\sigma_0}{C}\right)^{\frac{1}{n_1+2}}. \tag{15}$$

Since the instantaneous strain is determined according to (14) by the relationship

$$\varepsilon_0 = \frac{s_0^{m_1+1}}{m_1+1}, \tag{16}$$

then the instantaneous strain curve $\sigma_0 - \varepsilon_0$ is, according to (15) and (16), not monotonic in nature. The maximal stress σ_{01} which the specimen can sustain under loading is determined from the condition $d\sigma_0/d\varepsilon_0 = 0$, hence

$$s_{01} = \left[\frac{n_1+1}{(n_1+2)C}\right]^{\frac{1}{n_1+1}}, \quad \sigma_{01} = Cs_{01}^{n_1+2}, \quad \varepsilon_{01} = \frac{s_{01}^{m_1+1}}{m_1+1}.$$

The expression for the limit strain ε^* is determined by using (12) and (13), and depends on the values of the two combinations of exponents in (13)

$$L_1 = (m_2 - n_2 - 1) \text{ and } L_2 = (m_2 - n_2 + n_1 + 1).$$

The quantities L_1 and L_2 cannot equal zero simultaneously.

For $L_1 \neq 0$ and $L_2 \neq 0$

$$\varepsilon^* = \frac{1}{m_1+1}\left(\frac{\sigma_0}{C}\right)^{\frac{m_1+1}{n_1+2}} + \frac{(n_1+2)C}{DL_1L_2}\left(\frac{\sigma_0}{C}\right)^{\frac{L_2}{n_1+2}} - \frac{\sigma_0 s_0^{L_1}}{DL_1} + \frac{Cs_0^{L_2}}{DL_2}. \tag{17}$$

For $L_1 = 0$ we obtain

$$\varepsilon^* = \frac{1}{m_1+1}\left(\frac{\sigma_0}{C}\right)^{\frac{m_1+1}{n_1+2}} + \frac{\sigma_0}{D}\ln\left[\frac{1}{s_0}\left(\frac{\sigma_0}{C}\right)^{\frac{1}{n_1+2}}\right] - \frac{C}{DL_2}\left[\left(\frac{\sigma_0}{C}\right)^{\frac{L_2}{n_1+2}} - s_0^{L_2}\right].$$

For $L_2 = 0$ we obtain

$$\varepsilon^* = \frac{1}{m_1+1}\left(\frac{\sigma_0}{C}\right)^{\frac{m_1+1}{n_1+2}} + \frac{\sigma_0}{DL_1}\left[\left(\frac{\sigma_0}{C}\right)^{\frac{L_1}{n_1+2}} - s_0^{L_1}\right] - \frac{C}{D}\ln\left[\frac{1}{s_0}\left(\frac{\sigma_0}{C}\right)^{\frac{1}{n_1+2}}\right].$$

Presented in the figure as an illustration are the dependences $\varepsilon_0(\sigma_0)$ and $\varepsilon^*(\sigma_0)$ for C = D = 1, $n_1 = 2$, $m_1 = 6$. We examine two combinations of values of the remaining exponents characterized by the parameter k. In the case k = 1 the exponents m_2 and n_2 take the values $m_2 = 9$ and $n_2 = 7$, while in the case k = 2 they are $m_2 = 7$ and $n_2 = 9$. The curve $\varepsilon_0(\sigma_0)$ which is common to both cases is superposed by solid lines, while the curves $\varepsilon^*(\sigma_0)$ for k = 1 and 2 are superposed, respectively, by dashed and dash-dot lines. It is evident from (17) and Fig. 1 that the dependence $\varepsilon^*(\sigma_0)$ is not monotonic for k = 1 and decreases monotonically for k = 2.

Let us note that the nonmonotonicity of the function $\varepsilon^*(\sigma_0)$ results from the relationship of the exponents for components governing the creep characteristics of the material. The nonmonotonicity appears when the creep rate has a higher degree of dependence on the stress as compared with the dependence of the cumulative creep damage. In this case taking account of the nonlinear instantaneous characteristics plays a part analogous to that of the nonmonotonicity noted earlier when using different functional dependences for the creep rate and the cumulative damage rate [4].

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STEADY-STATE CREEP IN REFRACTORY COMPOSITES

V. V. Dudukalenko and S. P. Shapovalov UDC 539.376

There are anomalous variations in creep rate in the range 1000-1800°C for corundum-based refractories having high concentrations of ZrO_2 inclusions.

Measurements have been made [1] on torsion on cylindrical specimens under conditions of steady-state creep at various temperatures. The characteristic dependence of the creep rate on load (Fig. 1) shows that there is an anomalous viscosity change in the region of the phase transition temperature of ZrO_2 [1].

1. We use a form of the Bingham model to approximate the relationship shown in Fig. 1. The dissipative functions D are taken as differing from small and large strain rates ε_{ij} and for the corresponding stresses σ_{ij} [2]:

$$D = k\sqrt{\varepsilon_{ij}\varepsilon_{ij}} + (1/2)\nu\varepsilon_{ij}\varepsilon_{ij},\ s_{ij} = \sigma_{ij} - (1/3)\sigma_{ll}\,\delta_{ij} = \partial D/\partial\varepsilon_{ij},$$

where k is the plasticity limit and ν is the viscosity, while the subscript a in k_a and ν_a denotes those quantities for high strain rates.

The invariants $\gamma = \sqrt{\varepsilon_{ij}\varepsilon_{ij}}$, $\tau = \sqrt{s_{ij}s_{ij}}$ always simultaneously act as symbols for the shear components in torsion. In the (ρ, φ) polar coordinate system, the maximal stresses are obtained at the surface of a rod $\rho = R$, while the transition from $\tau = k + \nu\gamma$ to $\tau = k_a + \nu_a\gamma$ may occur at $\rho = R_1$. The zone of rapid creep propagates towards the center of the rod as the stresses increase. It can be shown that the piecewise-linear $\tau(\gamma)$ relationship goes over to a smooth conjugate one for the dependence of the torsional moment M on the torsion rate θ. As $\gamma = \theta\rho$, we have

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